Objectives
Density of Rocks
Density with depth
Density measurement
Reading
EOMA

Gravity 6
Density

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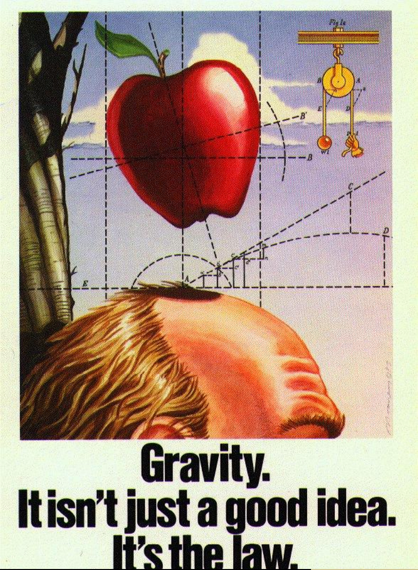
Potential Fields Geophysics: Week 6

Objectives for Week 5



- Bulk, true, and natural density
- Density of the Earth
- Density measurements

Density and compounds

Densities of rocks near the surface of the Earth range from about 800 kg m^{-3}–3000 kg m^{-3}, and depend on factors like composition, crystallinity, void fraction and water saturation. Often the "true density" refers to the mass per unit volume with no voids present. The "bulk density" refers to mass per unit volume of a dry rock, including void space. The "natural density" or "saturated bulk density" refers to the mass per unit volume where the void space is filled (saturated) with water or another fluid.

The true density can be estimated from the components of the rock, if the molecular formulae for the components are known, together with their volume fraction:

$$\rho = \sum_{i=1}^{N} \frac{X_i M_i}{V_i}$$

where: X_i is the mole fraction of component i and is dimensionless. M_i is the molecular mass (also called the molar mass) of component i and is usually expressed in units of g / mol. V_i is the fractional volume of component i and is usually expressed in units of m^3 / mol. N is the total number of components in the rock.

For each component, the effect of pressure and temperature on density can be estimated based on the isothermal compressibility and coefficient of thermal expansion, respectively.

$$V_i(X, P, T) = \bar{V}_i + \frac{\partial \bar{V}_i}{\partial P} P + \frac{\partial \bar{V}_i}{\partial T}(T - 1673)$$

where:

$\bar{V}_i$ is the partial molar volume of component i at 0.0001 GPa pressure and 1673 K

$\frac{\partial \bar{V}_i}{\partial P}$ is the coefficient of isothermal compressibility of component i (how the molar volume changes with pressure at constant temperature) (m^3/mol GPa).

$\frac{\partial \bar{V}_i}{\partial T}$ is the coefficient of thermal expansion of component i (how the molar volume changes with temperature at constant pressure) (m^3/mol K).

Macroscopic density

At a macroscopic level, one can characterize the true density of a rock from its mineral, glass, lithic or organic constituents. That is:

$$\rho = \sum_{i=1}^{N} \rho_i v_i$$

where ρ_i and v_i are the density and volume fraction of phase i, respectively.

Example

Consider a dense basaltic rock with a modal mineral composition determined by point count of:

Mineral	volume fraction
plagioclase	0.45
pyroxene	0.4
olivine	0.04
opaques	0.08
other	0.03

Where opaque minerals are some combination of magnetite and ilmenite and other is likely alteration (clay) minerals. Prove to yourself that the true density of this rock is approximately 3150 kg m^{-3}.

Phase	Density (kg m^{-3})
plagioclase	2690
K-feldspar	2500–2600
quartz	2650
pyroxene	3300–3360
olivine	3320
calcite	2710
magnetite	4890
ilmenite	4790
clay minerals	2500–2600
basaltic glass	2300 – 2700
obsidian	2300–2600
anthracite	1300–1500
bitumin	1100 –1300

Some common mineral, glass and organic constituents of rocks and their common densities. Note that these densities can vary substantially depending on exact composition.

Density units

Density is reported here in SI units (MKS). Often in geophysics, density is reported as g cm^{-3} because gravity is normally reported in mGal, where a Gal is 1 cm s^{-2}. Just remember to do the unit conversion!

Density and porosity

Porosity, of course, changes the bulk density of most rocks. Porosity includes any void, such as intergranular space, fracture, or bubble. If the pore space is dry then, neglecting the density of air:

$$\rho_{bulk} = \rho(1 - v_p)$$

where ρ_{bulk} is the bulk density of the dry porous rock, ρ is the "true density", which can be estimated from the constituents without considering pore space, and v_p is the volume fraction of pore space.

Example

Suppose the basalt discussed on the previous slide ($\rho = 3150\,\mathrm{kg\,m^{-3}}$) has a porosity of 30%. Prove to yourself that the bulk density of this rock is approximately $\rho_{bulk} = 2200\,\mathrm{kg\,m^{-3}}$.

Lithology	fractional porosity
Unconsolidated deposits	
gravel	0.25–0.4
sand	0.25 – 0.5
silt	0.35 – 0.5
clay	0.4 – 0.7
tephra	0.4 – 0.75
Rocks	
dense crystalline rock	0–0.05
fractured crystalline rock	0–0.1
shale	0–0.1
limestone	0–0.3
sandstone	0.05–0.3
karst limestone	0.25–0.6
fractured basalt	0–0.5
pahoehoe lava	0.2 –0.5

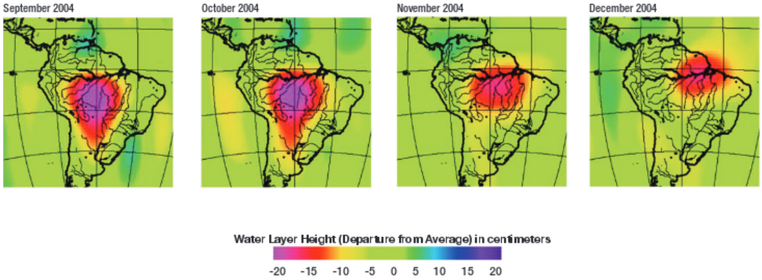
This table makes it quite clear that the density of rocks in the near-surface environment is strongly controlled by their porosities, which are quite variable even for a given lithology.

Gravity and lithology

Primarily because porosity is quite variable, there is a weak correlation between gravity anomalies and specific lithologies.

Saturated bulk density

Density and gravity change with water saturation. One function of gravity data is to monitor regional groundwater and surface water levels – by equating the change in gravity with change in water distribution, essentially the mass of water present in the near-surface. These maps show GRACE derived water data for the Amazon basin, indicating low water levels throughout the basin compared to average conditions. The huge advantage of GRACE acquisition is that repeated gravity maps are made of the same area and so differences in water level can be tracked.



The density of a saturated or partially saturated rock is:

$$\rho_{sat} = \rho(1 - v_p) + \rho_f v_f$$

where ρ_f is the density of the fluid and v_f is the volume fraction of fluid. Note that $v_p \geq v_f$.

Example

Suppose a quartz sand (2650 kg m^{-3}) has 30% porosity and is saturated with a hydrothermal brine (1125 kg m^{-3}). Prove to yourself that the density of this saturated rock is 2192.5 kg m^{-3}.

Change in density with depth in basins

Given the strong relationship between density and porosity (saturated or unsaturated), it is not surprising that the densities of some lithologies change significantly with depth. This is most dramatic in a sedimentary basin, where the sediments at the surface are likely unconsolidated and lithify with depth due to compaction and diagenesis. The change in density with depth in sedimentary basins is crucial to understand in interpreting gravity data, especially because so any economic and academic applications of gravity methods involve understanding the geometries and depths of sedimentary basins.

Various authors have developed functional relationships to model the expected change in basin density contrast with depth. One model is:

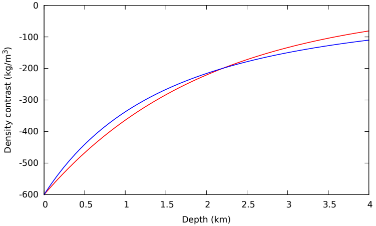
$$\Delta\rho(z) = \frac{\Delta\rho_o^3}{(\beta z - \Delta\rho_o)^2}$$

where: $\Delta\rho(z)$ is the change in density contrast with depth, $\Delta\rho_o$ is the density contrast at the surface, β is an attenuation factor that governs the change in density with depth and z is depth.Typical values used by Chakravarthi and Sundararajan (2007) are $\Delta\rho_o = -600\,\mathrm{kg\,m^{-3}}$, and β =100–200 $\mathrm{kg\,m^{-3}\,km^{-1}}$.

Another model for the change in density contrast with depth in a sedimentary basin is the exponential model, used by Garcia-Abdeslem and others:

$$\Delta\rho(z) = \Delta\rho_o \exp(-\alpha z)$$

with $\alpha = 0.5$, approximately, and z is depth in kilometers.



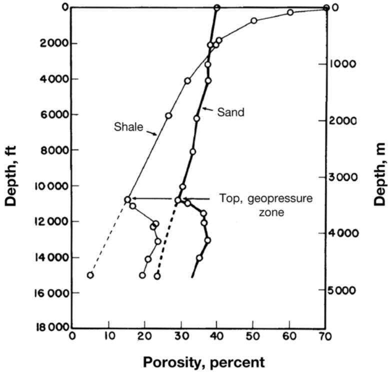
The power law model of Chakravarthi and Sundararajan (2007) is shown in blue and the exponential model in red. Clearly these models can produce similar results. Ideally, either model is fit to actual measurements of the change in density contrast with depth.

Role of overpressure in basin porosity and density

Basin porosity or density is often observed to change in a roughly exponential or power-law fashion with depth. Nevertheless, several features of basins can cause significant deviation from this general rule. For example, there are abrupt changes in lithologies with depth in some basins.

In deep sedimentary basins, overpressure of pore fluids at depth can lead to *increased* porosity with depth. High pore pressure can occur where porous and permeable water-filled sediments are buried by low permeability or impermeable sediment, such as clay, trapping fluids in an otherwise porous sedimentary section. The pore pressure in such sedimentary sections increases with deeper burial. High pore pressure can lead to an "under-compacted" zone, with relatively high porosity and low density. This is a common situation in the Gulf of Mexico, where sediments can be buried rapidly by relatively impermeable clays.

Another source of high pore pressure, and relatively high porosity and low density at depth, occurs in geothermal systems, where fluids can be heated, expand, and increase pore pressure.



These Gulf of Mexico profiles show the change in porosity with depth in a section dominated by shale and be sandstone. The excess pore pressure associated with impermeable horizons, here labeled the top of the geopressure zone, causes abrupt decrease in density with depth.

Change in density with depth in the lithosphere

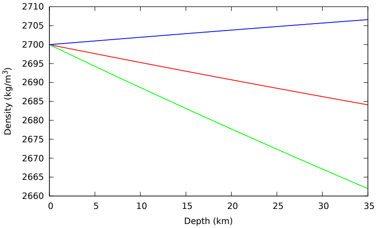
Many factors control the change in density with depth in the lithosphere, including change in composition, temperature and pressure. We can characterize the change in density with pressure and temperature in terms of compressibility and thermal expansion, respectively. These are related to density as:

$$\alpha = -\frac{1}{\rho}\frac{\partial \rho}{\partial T}, \ \beta = \frac{1}{\rho}\frac{\partial \rho}{\partial P}$$

Where α is the coefficient of thermal expansion and β is the compressibility. Gerya (2010) gives a simplified equation of state for dependence of lithosphere density on pressure and temperature conditions as:

$$\rho = \rho_r \exp\left[\beta(P - P_r) - \alpha(T - T_r)\right]$$

where ρ_r, P_r, and T_r are the reference density (e.g., 2700 kg m^{-3}), pressure (e.g.,10^5 Pa) and temperature (e.g., 298 K), respectively. This equation is a simplification because the compressibility and coefficient of thermal expansion themselves depend on pressure and temperature, and because the pressure, P, depends on the density of the overlying rocks, which is variable with depth. Nevertheless, the equation gives a good sense of the variation in density within the crust as a function of P and T conditions. The three curves are plotted by solving the equation of state at left, assuming a coefficient of thermal expansion of 5×10^{-5} K^{-1} and compressibility of 5×10^{-11} Pa^{-1}.



The three plotted lines are change in density with depth for a geothermal gradient of 35°C km^{-1} (green), 30°C km^{-1} (red), and 25°C km^{-1} (blue). For relatively high geothermal gradients, the equation of state predicts a decrease in density with depth because the coefficient of thermal expansion dominates. For a low geothermal gradient, the equation of state predicts an increase in density with depth, because compressibility dominates.

Seismic velocity and density

From early on, models of the change in density with depth for terrestrial planets have relied on the relationship between seismic velocity and density (e.g., Jeffreys, 1937). Compressional (V_p) and shear (V_s) wave velocities are:

$$V_p = \left[\frac{k + 4/3\mu}{\rho}\right]^{1/2}$$

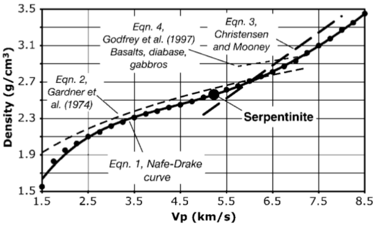
$$V_s = \left[\frac{\mu}{\rho}\right]^{1/2}$$

where k is the bulk modulus, and μ is the shear modulus. Solving these two equations for μ and equating them yields:

$$\rho = \frac{k}{V_p^2 - \frac{4}{3}V_s^2}$$

The direct application of this relationship to the Earth is hampered by several factors, such as an approximately power law relationship between the bulk modulus and density, and effect of numerous other factors, such as effect of temperature, presence of fluids, and the like on seismic wave velocity.

This relationship is well supported by the Drake-Nafe curve, which is an empirical relationship between measured compressional wave velocity and measured density. Other authors have derived similar relationships, all of which have considerable uncertainty, summarized by Brocher (2005). In graphical form:



The filled circles show handpicked values by Brocher from the curve published by Ludwig et al. (1970). The solid line is the Nafe–Drake curve, a polynomial regression to these picks and is the preferred density versus V_p relation. See Brocher (2005) for the polynomial equations used to plot these curves.

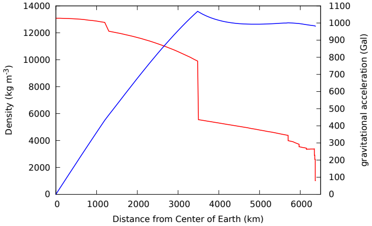
Density of the whole Earth

For the whole Earth, we can write the change in density with radius as:

$$\frac{d\rho}{dr} = \frac{\partial \rho}{\partial P}\frac{dP}{dr} + \frac{\partial \rho}{\partial T}\frac{dT}{dr} + \frac{\partial \rho}{\partial \phi}\frac{d\phi}{dr} + \frac{\partial \rho}{\partial c}\frac{dc}{dr}$$

that is, density is a function of pressure (P), temperature (T), mineral phase (ϕ) and composition (c). Porosity is ignored for the deep Earth and is assumed to approach zero in the lower crust.

Understanding the density structure of the Earth is a matter of understanding the change in pressure, temperature, mineral phase and composition as a function of depth.The preliminary reference Earth model (PREM) of Dziewonski and Anderson (1981) attempts to take these factors into account drawing on a wide array of geophysical, mineral physics and geochemical data. Earth density (red curve) varies in a complex way with depth. Significant jumps can be seen within the lithosphere and across the core–mantle boundary (3480 km) and to a lesser extent across the outer–inner core boundary (1221 km) due to changes in composition and phase. With this density curve, we can now predict the Earth's gravity (blue curve) accurately with depth. Because of these density changes, gravity does not decrease linearly with depth, as we would expect with a homogeneous Earth, but reaches a maximum near the core–mantle boundary.



Example

Prove to yourself that gravitational acceleration at the core–mantle boundary ($r = 3480$ km) is about 1060 Gal, for an average core density of 10900 kg m^{-3}.

Density measurement of rock samples

Archimedes' Principle can be used to determine the density of rock samples. Buoyancy force arises because of pressure increases with depth and because pressure acts on all sides of a submerged object, such as a rock in a tank of water. The buoyancy force is equal to the weight of the fluid displaced by the object:

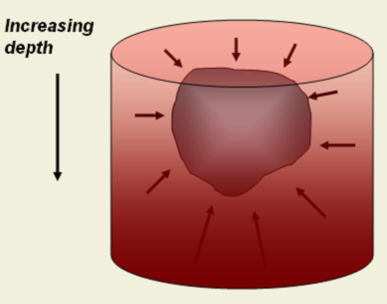
$$F_b = \rho_f V_r g$$

where ρ_f is the density of the fluid, V_r is the volume of the rock, and g is gravity. With some algebraic substitution, this can be solved for the density of the rock:

$$\rho_r = \frac{W_a}{W_a - W_f}\rho_f$$

where W_a and W_f are the weight of the sample in air and fluid (water), respectively, and ρ_f is the density of the fluid.

In practice, a sample is weighed by suspending it from a scale, then submerged fully and weighed in water, to determine its density. This usually gives a bulk density (unsaturated) because air remains in the void space. To assure this, samples are often dried in a low temperature oven and very porous samples are covered in paraffin wax to seal the pores. Usually a large number of samples (10 –30) are collected from each formation in a field area to constrain variation in the formation bulk density.

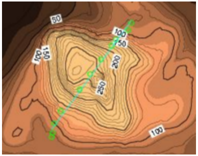


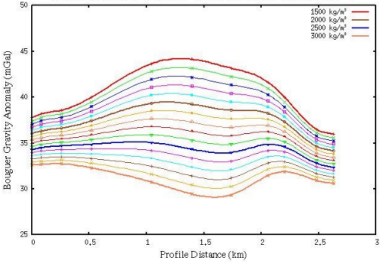
A rock submerged in water or another fluid experiences buoyancy force. Pressure on the object (illustrated schematically by the black arrows) is greater at depth in the container than at shallow depth ($P = \rho g h$, where h is depth in the container). The greater pressure at depth results in upward force on the rock, decreasing its weight in water.

The Nettleton Method

L. L. Nettleton, working in the 1930s, realized that in an area is homogeneous in density, no Bouguer anomaly variations should result after data reduction of a gravity survey carried out over a topographic feature. Therefore, gravity readings collected across such topography can be used to estimate the bulk density, or saturated bulk density, of the uniform lithology in the survey area. The Bouguer density that causes the minimum correlation between topography and gravity is the best estimate of the bulk or saturated formation density. His method involves collection of gravity readings across gently varying topography – the gentle topography minimizes uncertainties associated with terrain corrections made in steep topography. Some topography is required to see the variation in gravity with elevation and to find the density that minimizes the correlation. Uncertainty in the method occurs because the geology may not be homogeneous, or there may be regional gravity gradients that obscure the correlation.

This gravity survey done by Magnús Guǒmundson and Rémy Villeneve over a small hyaloclastite ridge near Reykjavik, Iceland, illustrates the method. The map shows the position of gravity readings collected across a hill. The gravity data are processed using different Bouguer densities to search for the minimum correlation with topography (the set of curves at right).





In this case they found low correlation for Bouguer densities between 2100–2600 kg m^{-3}, with lowest correlation around 2500 kg m^{-3}.

The Parasnis Method

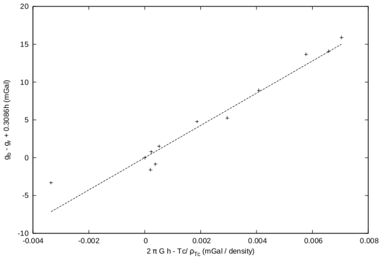
D. S. Parasnis expanded on the Nettleton method. He realized the linear equation of the Bouguer anomaly formula can be rearranged such that the slope of the line is the density, given a homogeneous terrain:

$$g_L - g_r + dg_{FA} = dg_B - dg_{Tc}$$

where g_L is the latitude corrected gravity, g_r is the reference gravity, usually gravity measured at a base station, dg_{FA} is the free air correction, dg_B is the Bouguer correction, dg_{Tc} is the terrain correction. To implement the Parasnis method, let:

$$\begin{aligned} x &= 2\pi G h - \frac{T_c}{\rho_{Tc}} \\ y &= g_L - g_r + 0.3086h \end{aligned}$$

where h is the height difference between the reference gravity station, g_r, and the latitude-corrected gravity station g_L, T_c is the terrain correction, and ρ_{Tc} is the initial density used in the terrain correction. Note the units of x are mGal/density and the units of y are mGal, meaning the slope of the line through a set of x, y points is the density. A set of x, y points calculated using these equations for the Guŏmundson and Villeneve data discussed on the previous slide is plotted at right.



Example

Using a reference gravity value of 16.1 mGal, reference elevation of 86.9 m, and $T_c = 0$, calculate the x, y pair for the Parasnis density model, for a gravity station: $g_L = -18.27$ mGal, elevation = 243.91 m. Prove to yourself that for this gravity station, $x = 0.00658, y = 14.08$. For the Iceland data shown, the density is $\rho = 2131 \pm 125$ kg m^{-3} after a line is fit to all of the x, y pairs.

Additional Reading

The book by Hinze et al. (2013) has an excellent overview of density and its role in geophysical exploration. Gerya (2010) has an excellent introductory discussion of the equation of state for density as a function of pressure and temperature in the Earth. This discussion is expanded upon in Anderson (1989) and Anderson (2007).

- Anderson, D. L. (2007). New theory of the Earth. Cambridge University Press.
- Anderson, D. L. (1989). Theory of the Earth. Cambridge University Press.
- Gerya, T. (2010). Introduction to numerical geodynamic modeling. Cambridge University Press.
- Hinze, W. J., von Frese, R. R., & Saad, A. H. (2013). Gravity and Magnetic Exploration: Principles, Practices, and Applications. Cambridge University Press.

Additional key references for this module are:

- Brocher, T. M. (2005). Empirical relations between elastic wavespeeds and density in the Earth's crust. Bulletin of the Seismological Society of America, 95(6), 2081–2092.
- Chai, Y., & Hinze, W. J. (1988). Gravity inversion of an interface above which the density contrast varies exponentially with depth. Geophysics, 53(6), 837–845.
- Chakravarthi, V., & Sundararajan, N. (2007). 3D gravity inversion of basement relief – A depth-dependent density approach. Geophysics, 72(2), I23–I32.
- Dziewonski, A. M., & Anderson, D. L. (1981). Preliminary reference Earth model. Physics of the Earth and Planetary Interiors, 25(4), 297–356.
- Emerson D.W. (1990) Notes on mass properties of rocks: density, porosity, permeability. Exploration Geophysics 21 , 209–216.
- Garcia–Abdeslem, J. (1992). Gravitational attraction of a rectangular prism with depth-dependent density. Geophysics, 57(3), 470–473.
- Jeffreys, H. (1937). The density distributions in the inner planets. Geophysical Journal International, 4(s1), 62–71.

End of Module Assignment

1. A limestone karst aquifer has 30% porosity. In the dry season the water table is located at 20 m depth. In the wet season the water table is located at 2 m depth. (1) What is the change in density of this rock from unsaturated to saturated conditions? (2) What is the expected change in gravity at a station located in the middle of the karst aquifer from the dry season to the wet season, for the given water table depths. Explain your model and assumptions.

2. Brocher (2005) published a polynomial form of the Drake–Nafe curve for sediments and sedimentary rocks:

$$\rho = 1.6612V_p - 0.4721V_p^2 + 0.0671V_p^3 - 0.0043V_p^4 + 0.000106V_p^5, 1.5 < V_p < 6.1$$

   where V_p is in $\mathrm{km\,s^{-1}}$ and ρ is in $\mathrm{g\,cm^{-3}}$. Note that although this relation appears to be quite precise, in reality there is huge scatter in this relationship in reality. Assume a basin is characterized by a linear change in P-wave velocity with depth ($V_p = 1.5 + 0.5z$, $0 < z < 5$ km). Use the polynomial to estimate the change in density with depth and the density contrast between basin sediments and the surrounding bedrock, as a function of depth (density contrast between the basin and the bedrock at 5 km is zero). Graph your results and provide a brief explanation of the plot.

EOMA

5. The following table contains gravity and elevation data collected across a hill. Estimate the density of the hill using the Parasnis' variation on the Nettleton method, by fitting a line to the calculated x and y values derived from the gravity data. Some gravity reductions have already been done, so you will use these equations and regress on x and y to find the density:

$$x = 2\pi G h - \frac{T_c}{\rho_{Tc}}$$

$$y = g_s - g_r + 0.3086h$$

where h is the relative elevation, T_c is the terrain correction, $\rho_{Tc} = 2000\,\mathrm{kg\,m^{-3}}$ is the density used in the terrain correction, g_s is the station gravity, g_r is the reference gravity (use the base station), G is the gravitational constant.

Station ID	relative gravity (mGal)	relative elevation (m)	terrain correction (mGal)
base	100.00	0	0.00
sta1	91.67	3.11	6.4
sta2	90.15	6.28	7.12
sta3	90.41	13.44	5.66
sta4	92.92	21.95	2.10
sta5	93.43	25.94	1.00
sta6	83.94	31.00	8.0
sta7	84.92	27.16	7.85
sta8	90.78	11.80	5.65
sta9	94.1	6.46	3.8
sta10	96.66	4.75	1.95