Magnetics

Introduction to Filtering using the 2D Fourier Transform
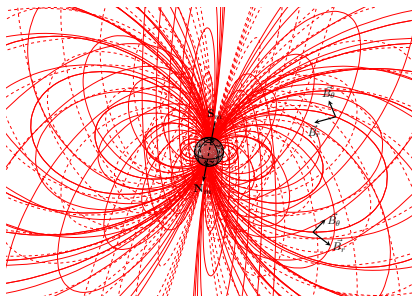
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Potential Fields

Schematic Earth dipolar magnetic field. The field lines placed in the page plane are drawn as thick lines, those back with dashed lines and the field lines in front of the page with thin lines.

- Learn about the 2D Fourier transform

- Learn about filtering maps

- Learn about the radial power spectrum

- Use scripts to filter magnetic map data

The one-dimensional Fourier transform is used to transform any function from the spatial (or time) domain into the wavenumber (or frequency) domain. This transform is extremely useful in geophysics because of the ease of convolving functions and filtering data in the wavenumber domain. The two-dimensional Fourier transform is equally useful for transforming and filtering map data, for example the magnetic field as a function of distance east and north. The 2D forward and inverse Fourier transform is:

$$F(k_x, k_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \exp(-i2\pi(k_x x + k_y y)) dx dy$$

$$f(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(k_x, k_y) \exp(i2\pi(k_x x + k_y y)) dk_x dk_y$$

where:

k_x and k_y specify the 2D wavenumber, in x and y directions,

$f(x, y)$ is the function (e.g., change in magnetic field as a function map location, x, y,

and i is the complex conjugate.

$f(x, y)$ is a function in space, $F(k_x, k_y)$ is the same function in the wavenumber domain. For the discrete 2D Fourier transform

$$F(k_x, k_y) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) \exp\left[-i2\pi\left(\frac{k_x x}{M} + \frac{k_y y}{N}\right)\right] dx dy$$

$$f(x, y) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} F(k_x, k_y) \exp\left[i2\pi\left(\frac{k_x x}{M} + \frac{k_y y}{N}\right)\right] dk_x dk_y$$

As with the 1D case, the 2D Fourier transform is computed with a fast Fourier transform (FFT) algorithm.

The magnitude of the amplitude spectrum of a 2d image is found from the real and imaginary components of its Fourier transform:

$$|F(k_x, k_y)| = \sqrt{\text{Re}(k_x, k_y)^2 + i\text{Im}(k_x, k_y)^2}$$
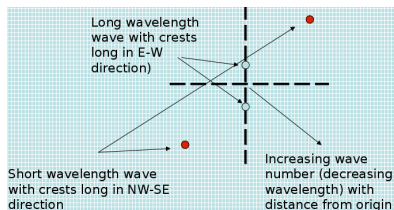
and the power spectrum is

$$P(k_x, k_y) = \text{Re}(k_x, k_y)^2 + i\text{Im}(k_x, k_y)^2$$

Note that:

$$P(k_x, k_y) = P(-k_x, -k_y)$$

This means a graph of the 2d power spectrum, plotted for $-M/2 < k_x < M/2$ and $-N/2 < k_x < N/2$ has two-fold symmetry – the same pattern occurs after $180°$ rotation of the 2d power spectrum. The wavenumber increases with increasing distance from the origin, and the wavelength decreases with distance from the origin.

A 2d power spectrum shows the wavelengths at which signals are most prominent on the map. The direction associated with each wave (sine and cosine term of a given wavelength) is also shown on the power spectrum as illustrated in this figure.

Long wavelength wave with crests long in E-W direction)

Short wavelength wave with crests long in NW-SE direction

Increasing wave number (decreasing wavelength) with distance from origin

The light blue circles mark a wavenumber, $(0, k_y)$, that corresponds to a relatively long wavelength signal that undulates in a N–S orientation on the original map, $f(x, y,)$. That is, the crests and troughs of the wave are oriented E–W. The red circles mark a wavenumber, (k_x, k_y), that corresponds to shorter wavelength signal with crests and troughs trending NW–SE.

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Objectives

The 2D Fourier Transform

Radial power spectrum

Band-pass

Upward continuation

Directional Filters

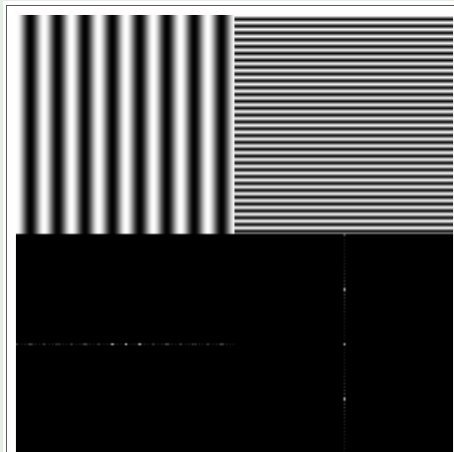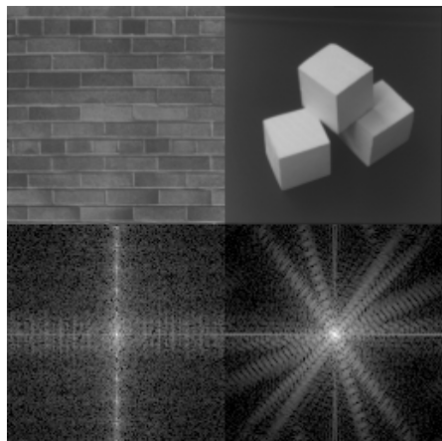Vertical Derivative

RTP

Additional Resources

EOMA

It takes practice to get used to looking at 2D power spectra, but these images are full of information about any map or image. Consider these examples. Two waves are shown in the images at top, one with crests and troughs oriented N–S at relatively long wavelengths and the other with crests and troughs oriented E–W at relatively short wavelengths.

Their Fourier transforms are shown in the lower panels for each image. Note that almost all of the 2D power spectrum is black, indicating that the amplitudes for these k_x, k_y spectra are close to zero. Large amplitude spectra, indicated by the "bright" spots, correspond to the wavelength and orientation of each period wave.

Also note that there is a "bright" spot at (0,0). This corresponds to the mean value of the map. Just like in 1D FFTs, any trends on the map must be removed before the Fourier transform of the map is calculated. Also, you can see some ringing in each 2D spectrum associated with the wave repeating at multiple wavelengths.
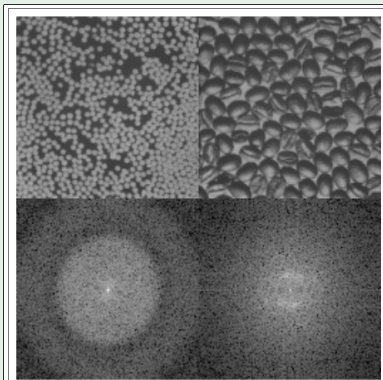
Examples

These pairs of images and their 2D power spectra illustrate the ease with which 2D power spectra can be used to identify lines, edges, or trends in data. For the brick pattern, the largest amplitude spectra are located at wavelengths corresponding to the size of the bricks and in "N–S" and "E–W" directions. For the randomly oriented blocks, the power spectrum shows prominent trends in the image associated with block edges.

Consider how you could filter these images. If you only wanted to see the brick edges, and no other textures on the image, you could zero all spectra except the largest amplitude along the axes. You could filter the block image to only see particular edges by zeroing spectra as a function of direction. These are box filters, and work exactly the way box filters work in 1D.

Examples

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Consider how the 2d power spectrum is affected by particle shape. The spherical white particles create a very symmetric power spectrum. The rapid change in power spectra amplitudes at relatively short wavelengths coincides with the diameter of the particles. The coffee bean spectrum is less symmetric, as are the beans, but can also be used to characterize the size of beans. Since the sizes of the coffee beans vary, the power spectrum is more complicated. Put another way, the particle size distribution is reflected in the 2D power spectrum. Comparing the two 2D power spectra, it is easy to see that on average the coffee beans are larger than the spheres. Note that the power spectrum would basically look the same for any similar arrangement of spheres or beans, so the spectrum is a way quantify particle size and shape, many particles at a time.

Now consider a more complex image – the girl (the one on the left). Although the image is complicated, characterized by a relatively diffuse 2D power spectrum, the prominent slanted lines in the image are easily seen in the power spectrum. The image of the baboon on the right is characterized by smoother, longer wavelength variation and the occurrence of prominent nearly vertical trends.
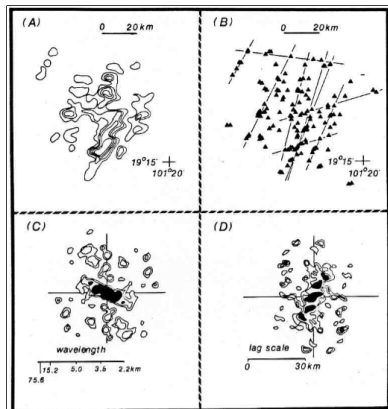
Examples

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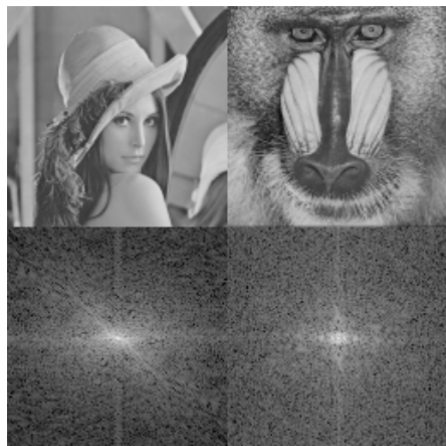
It is possible to analyze almost any sort of map using the 2D FFT and related methods. Consider the plot at right. (B) shows the distribution of volcanoes in part of the TransMexican volcanic belt (Mexico) as black triangles. Are there patterns in the distribution of these volcanoes, or are they distributed in a completely spatially random way?

The volcano location data were gridded and a 2D FFT was made of the grid, resulting in the 2D power spectrum shown in (C). Only spectra with amplitudes greater that the 90th, 95th, and 99th (black blotches) percentiles are contoured. These large amplitude spectra are anisotropic, with isolated spectra occurring at wavelengths as short as 9.5 km, indicative of the occurrence of NE-trending alignments in the original gridded data. The power spectrum can be filtered to only include these large amplitude spectra and transformed back into the spatial domain (A), showing these statistically significant NE-trending vent alignments. The 2D power spectrum is closely linked, mathematically, to the 2D autocorrelation function (shown in D) for these volcanoes) and the Hough transform, which shows the alignments drawn as lines in (B). The 2D FFT can be used to identify statistically significant features in maps, and can be used to filter data sets, just like filtering of 1D profiles.

(Figure from Connor, 1990, Cinder cone clustering in the TransMexican volcanic belt, JGR)

The power spectrum of maps is often summarized using the radial power spectrum. This is a graph on which the amplitudes of spectra are averaged, so we get an average change in the amplitude of spectra with distance from the origin – as a function of wavelength. Geophysical maps are usually characterized by large amplitude, long wavelength spectra and decreasing amplitudes at longer wavelength.
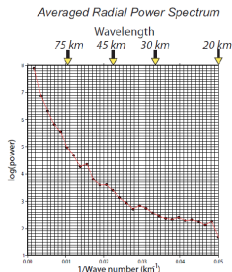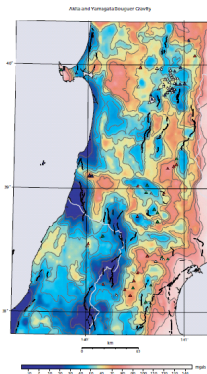

The map shows gravity anomalies in the Tohoku region, Japan, and the radial power spectrum for this map, plotted in log units of power, P. Note that the structure of the radial power spectrum is very compatible with one-dimensional filters that you have already seen. For example, low-pass filters are better visualized on the radial power spectrum than on the 2d spectrum. A disadvantage of the radial power spectrum is that directional properties of the map – in this case the elongate N–S-trending gravity anomalies associated with the volcanic arcs and fault-bounded basins – are not illustrated.

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The radial power spectrum can be used to low-pass, high-pass and band-pass filter potential fields maps, like the gravity map shown at left. Here a low-pass filter operation was done by:
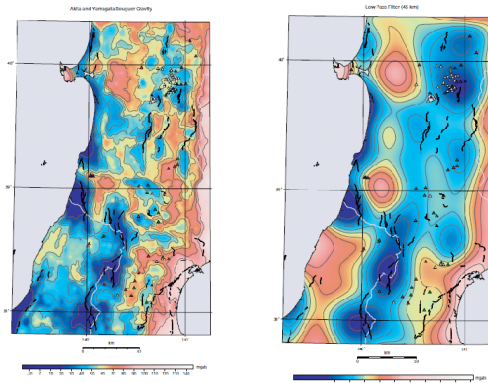
- gridding the gravity data so there is equal data spacing in the N–S and E–W directions
- removing a regional trend from the map grid
- calculating the forward 2D FFT of the map
- filtering the radial power spectrum to include only long-wavelengths (a linearly ramped low-pass filter was used)
- calculating the inverse 2D FFT
- putting the regional trend back in the map (optional)

The low-pass filtered map highlights long-wavelength features of the maps, especially the positive gravity anomaly associated with subduction on the eastern boundary of the map, the predominately negative gravity anomalies associated with thickening of the crust around clusters of arc volcanoes, and isolated positive anomalies in the back-arc.

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Upward continuation is the process of estimating the form of the gravity or magnetic field by measuring the field at a lower elevation and extrapolating upward, assuming continuity. It is achieved in maps using a filter that attenuates power spectra by an exponential function of wavelength, exactly analogous to the one-dimensional case.


The are composite aeromagnetic maps (consisting of data collected on many different surveys) for the state of Alaska. Upward continuation by 10 km greatly attenuates the map as a function of wavelength, creating a smoother variation in anomalies. At this scale, the upward continuation is useful for identifying regional patterns in magnetic anomalies, associated with major tectonic features of Alaska. Alternatively, the upwardly continued map can be subtracted from the original data to enhance shallow local anomalies. This method was used by Blakely and colleagues in their analysis of aeromagnetic data and faults in the Seattle region (see Blakely et al., 2002, Geological Society of America, Bulletin, 114:169–177).

The 2D power spectrum can be used to enhance or suppress anomalies associated with particular directions or trends. For example, suppose relatively short-wavelength NW–SE trending anomalies (e.g., red dots) are cut. This will enhance other features on the map.

Cut this region to enhance the E-W trending longer wavelength anomalies
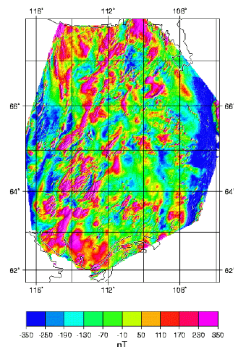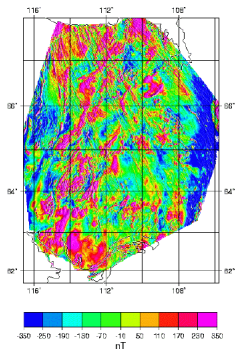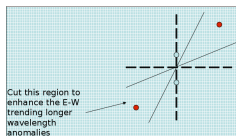
The aeromagnetic map of the MacKenzie dike swarm, Canada, shows numerous NNW-trending anomalies associated with very large igneous dikes. This dike swarm is the largest known on Earth, extending about 3000 km from the Arctic to the Great Lakes region in southern Canada and was formed approximately 1.2 billion years ago by the MacKenzie hot spot. Directional filtering of the magnetic map suppresses the relatively short-wavelength anomalies associated with the dikes. A residual map (the original map - the filtered map) would greatly enhance the magnetic anomalies associated with the major dikes.

The nth vertical derivative of a gridded potential field map, U, can be found by multiplying its Fourier transform by a simple filter

$$F\left(\frac{\partial^n U}{\partial z^n}\right) = F(U)|k_r|^n$$

where k_r is the radial wavenumber. So the 2nd vertical derivative is:

$$F\left(\frac{\partial^2 U}{\partial z^2}\right) = F(U)|k_r|^2$$

The 2nd vertical derivative is particularly useful for enhancing steep magnetic gradients (middle panel) and can be used with shaded-relief algorithms, just like those commonly applied to topographic data (right panel) to highlight features not easily seen in the original map (left panel).
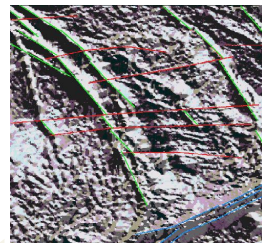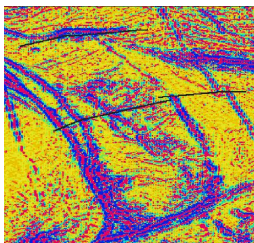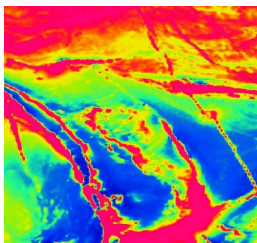
Reduction-to-pole is a method that uses the FFT to estimate the shape of a magnetic anomaly at the pole, assuming the direction of magnetization is known. This is accomplished with a direction cosines filter applied the the Fourier transform of the map:

$$R(k_r) = |k_r|^2 \frac{F(k_r) k_r^2}{B^2}$$

where

$$k_r = \sqrt{k_x^2 + k_y^2}$$

$$B = \frac{1}{|ik_x \cos I \cos D + ik_y \cos I \sin D + k_r \sin I|}$$

and I and D are the main field inclination and declination, respectively. The direction cosines in the north, east and vertical directions transform the field orientation.

Reduction To The Pole

Total Magnetic Intensity

Magnetic Fault Zone

Note the unusual form of the equation for B. The radial wavenumber, k_r is the distance (1/wavelength) from the origin on the 2D power spectrum. But the equation also depends on the distance "north" (k_x) and the distance "east" (k_y).

The goal of rtp is to simplify magnetic maps – essentially making them appear to be more like gravity maps. In practice, rtp is complicated because anomalies may be rotated by tectonism, rocks of different ages may be reverse or normally polarized and related factors. It is not always appropriate to filter an entire map, especially on a regional scale, with a single vector of magnetization. Nevertheless, rtp is an extremely useful method of map enhancement in some circumstances.
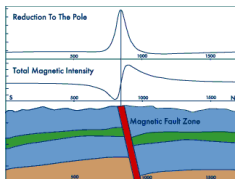
This module just scratches the surface of map filtering using the FFT. Additional methods include the pseudo-gravimetric transform, the analytical signal, and the total magnetic gradient. One of the best ways to get to know methods in map filtering is to actually do it! Use Generic Mapping Tools (GMT) to explore map filtering. One resource for processing geologic images with GMT is:

http://nbviewer.ipython.org/gist/dbuscombe-usgs/6238345 - a tutorial on using GMT and python to analyze geological images

Several articles in *The Leading Edge* about map filtering are particularly accessible. These include:

- Salem, A., Williams, S., Fairhead, J. D., Ravat, D., and Smith, R. (2007). Tilt-depth method: A simple depth estimation method using first-order magnetic derivatives. The Leading Edge, 26(12), 1502–1505.
- Verduzco, B., Fairhead, J. D., Green, C. M., and MacKenzie, C. (2004). New insights into magnetic derivatives for structural mapping. The Leading Edge, 23(2), 116–119.
- Li, X. (2008). Magnetic reduction-to-the-pole at low latitudes: Observations and considerations. The Leading Edge, 27(8), 990–1002.
- Li, X. (2003). On the use of different methods for estimating magnetic depth. The Leading Edge, 22(11), 1090–1099.

A historically important paper:

- Bhattacharyya, B. K. (1965). Two-dimensional harmonic analysis as a tool for magnetic interpretation. Geophysics, 30(5), 829–857.

Also refer to Hinze et al. (2013) pages 316–337 for a discussion of various filtering methods.

Let's do some map filtering! *san_raf_mag.dat* contains magnetic data collected over a small area in the San Rafael volcanic field, UT (USA), by undergraduates during the summer of 2011. The data were collected with a total field magnetometer and a GPS. Note that the average regional field (IGRF) has already been removed from these data. The UTM coordinates are given in WGS84, zone 12. We want to visualize these map data and do some basic map filtering with the 2D FFT to enhance magnetic anomalies.

1. Complete Steps 1–4 in the supplementary material. Be sure to plot a histogram of the magnetic data range, plot a map of point locations (filtered to delete bad data as described in the supplementary material) and make a good looking contour plot. Don't lose points - make informative captions!

2. Make an upward continuation of the magnetic data (Step 5). Try alternative values of the upward continuation and check out the map smoothing.

3. Reduce the magnetic data to the pole (Step 6). Use alternative values of the declination to try to enhance the map.

4. Prepare a short write-up discussing the magnetic anomalies on the map. Label prominent anomalies and discuss them in terms of the amplitude, wavelength, and shape. What can you infer about their remanent magnetization from the map? What can you infer about the origins of these anomalies?

5. (extra credit) Return to Blakely et al (2002) and note the process they used to filter their aeromagnetic data. Filter the map you created in assignment 3 (Seattle fault map) to try to enhance the anomalies.