Nature of Magnetic Fields

$\vec{B}$, $\vec{\mu}$ and $\vec{M}$

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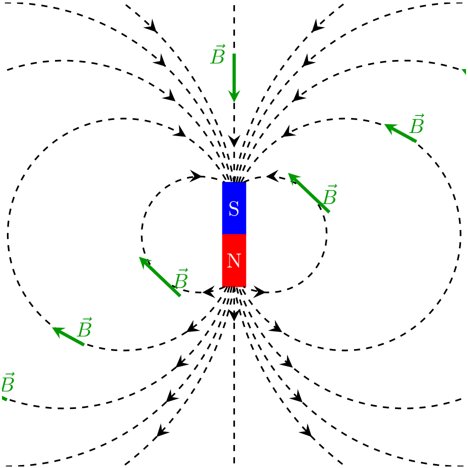
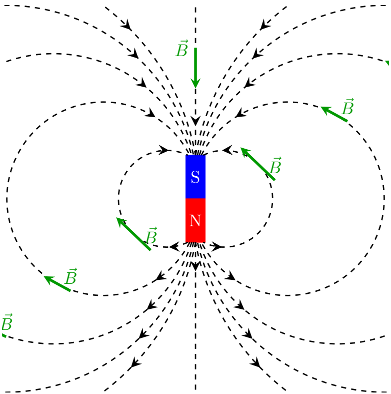
Potential Fields Geophysics

Objectives for this week



- Learn about magnetic fields and field strength
- Learn about the magnetic dipole moment
- Calculate the magnetic field around a dipole
- Learn about magnetization

magnetic field basics



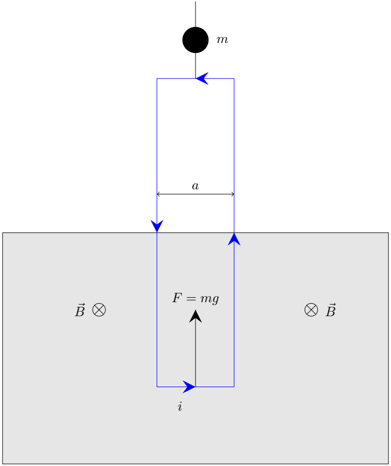
Magnets, magnetic rock, and electrical currents create magnetic fields in the space around them. The basic magnetic field vector, $\vec{B}$, describes the shape of the magnetic field in space induced by the presence of the magnet or other magnetic field creating object.

The dashed lines show the magnetic field lines, more properly but rarely called the magnetic induction lines. If one held a bar magnetic, like a compass, along one of these lines, it would orient parallel to that field line.

$\vec{B}$ is tangent to the field lines everywhere. The magnitude of $\vec{B}$ decreases as the field line density decreases and decreases with increasing distance from the magnetic object. That is, where magnetic field lines are close together, $\vec{B}$ is large. Where magnetic field lines spread apart, $\vec{B}$ is small.

The magnetic field lines are only a 2D representation of the magnetic field, which is really 3D. Sometimes geophysicists refer to magnetic field shells instead of magnetic field lines. Think of the figure as a cross-section through the magnetic shells.

units of $\vec{B}$



Consider an experiment. A rectangular wire circuit is created of width a and an electrical current i is passed through the circuit (blue). Then, one end of the wire circuit is placed in a uniform magnetic field (gray box), produced by a magnet. The field points into the page and has a magnitude B. Because the lower horizontal part of the wire is in the external magnetic field, and the upper horizontal part of the wire is not, a force is created on the wire. Given the directions of current flow and the external magnetic field, F is directed up. More mass, m (the black ball), must be added to prevent the wire from rising (assuming the wire is stiff enough). The magnetic force is created by charged particles (the electrical current) streaming through the magnetic field. The change in force due to application of the external magnetic field is

$$F = mg = iaB$$

$$B = \frac{mg}{ia}$$

The units of the magnitude of $\vec{B}$ are:

$$\frac{\text{kg m}}{\text{s}^2}\,\frac{1}{\text{amp m}}$$

units of $\vec{B}$

From the experiment described on the previous slide:

$$B = \frac{mg}{ia}$$

The current, i (amp) is the amount of electrical charge, q, that passes through the wire as a function of time, t. So,

$$i = \frac{q}{t}$$

$$B = \frac{F}{qv} = \frac{F}{ia}$$

The magnitude of the magnetic induction vector is directly proportional to the force F and inversely proportional to the charge times the charge velocity, v. In terms of units:

$$1 \frac{\mathrm{N}}{\mathrm{Amp\ m}} = 1\ \mathrm{T}$$

where T stands for Tesla. The SI unit of magnetic induction, commonly referred to as magnetic field strength, is Tesla.

Alternative units of $\vec{B}$

Since Earth's magnetic field is weak compared to 1 T, the unit nanoTelsa (nT) is commonly used in geophysics

$$1\ \mathrm{nT} = 1 \times 10^{-9}\ \mathrm{T}$$

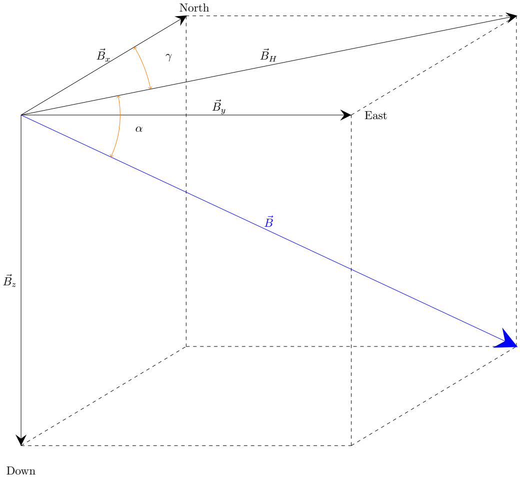
$$1\ \mathrm{nT} = 1 \times 10^{-3}\ \mu\mathrm{T}$$

Other non-SI units are used to describe magnetic field strength in geophysics, increasingly less commonly:

$$1\ \mathrm{nT} = 1\ \mathrm{gamma}$$

$$1\ \mathrm{nT} = 1 \times 10^{-5}\ \mathrm{gauss}$$

Orientation of $\vec{B}$



Like all vectors, think of the orientation of $\vec{B}$ in terms of its components in a coordinate system. Make the x component point north, the y component point east, and the z component point down.

$$B = \sqrt{B_x^2 + B_y^2 + B_z^2}$$

$$B_H = \sqrt{B_x^2 + B_y^2}$$

$$\alpha = \tan^{-1}\left[\frac{B_z}{\sqrt{B_x^2 + B_y^2}}\right]$$

$$\gamma = \sin^{-1}\left[\frac{B_y}{\sqrt{B_x^2 + B_y^2}}\right]$$

where α is the inclination of $\vec{B}$, and is positive down; γ is the declination of $\vec{B}$, and is positive east.

F and $\vec{B}$

Another way to think about the physical meaning of $\vec{B}$ is based on a magnetic version of Coulomb's Law, which describes the force that exists between two magnetic poles. The force is inversely proportional to the distance between the poles, as in Newton's Law of gravity. In SI units:

$$\vec{F}_m = C_m \frac{p_1 p_2}{r^2}$$

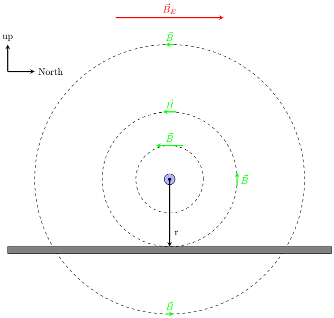
where $\vec{F}_m$ is the magnetic force between the poles (N), C_m is a proportionality constant (N/amp^2), p_1 and p_2 are the pole strengths (amp m), and r is the distance between the poles (m).
In SI units:

$$C_m = \frac{\mu_o}{4\pi}$$

where $\mu_o = 4\pi \times 10^{-7}$ N/amp^2, which is termed the magnetic permeability. $\vec{B}$ is the force acting at any point in space near a pole, p_1 supposing there is a pole of unit strength at the position of p_2.

$$\vec{B} = \frac{\vec{F}_m}{p_2} = \mu_o \frac{p_1}{r^2}$$

$\vec{B}$ near a wire - Ampere's Law



Ampere's law relates the magnitude of the magnetic field, $\vec{B}$, to the magnitude of an electrical current, i. Consider an electrical current carried by a wire flowing out of the page (blue circle).

A magnetic field is generated around the wire. $\vec{B}$ decreases with increasing distance, r from the wire. Ampere's Law gives:

$$\oint \vec{B} \cdot dl = \mu_o i$$

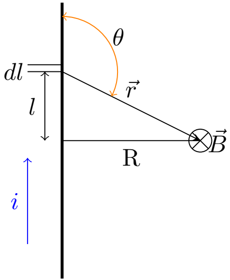
which in this case simplifes to

$$\vec{B} = \frac{\mu_o i}{2\pi r}$$

Magnetic Anomaly

Suppose you do a magnetic survey on the ground (gray box) moving under the electrical wire. The regional magnetic field, $\vec{B}_E$ is parallel to the ground surface. Prove to yourself that the electrified wire creates a positive magnetic anomaly as you traverse beneath it walking south to north. If $i = 30$ amp, and $r = 10$ m, prove the maximum anomaly is 600 nT.

Biot-Savart Law



The Biot-Savart Law provides a way to estimate the strength of the magnetic field associated with an electrical current. In terms of magnitude of the field only, the B-S Law states:

$$B = \int dB = \int \frac{\mu_o i}{4\pi} \frac{\sin\theta}{r^2} dl$$

Example

Consider a long straight wire

$$B = \int dB = \int \frac{\mu_o i}{4\pi} \frac{\sin\theta}{r^2} dl$$

$$r = \sqrt{l^2 + R^2}$$

$$\sin\theta = \frac{R}{\sqrt{l^2 + R^2}}$$

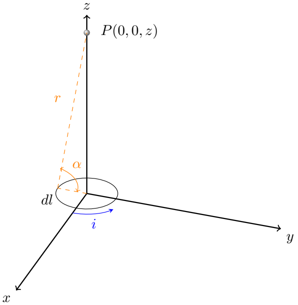
$$B = \frac{\mu_o i}{4\pi} \int_{-\infty}^{\infty} \frac{R}{(l^2 + R^2)^{3/2}} dl$$

Prove to yourself that it follows that

$$B = \frac{\mu_o}{2\pi} \frac{i}{R}$$

the same result as obtained from Ampere's Law.

$\vec{B}$ on the axis for a current loop



The Biot-Savart law provides a way to determine $\vec{B}$ on the z axis, normal to a current loop:

$$dB = \frac{\mu_o i \cos\alpha dl}{4\pi r^2}$$

$$B = \int \frac{\mu_o i \cos\alpha dl}{4\pi r^2}$$

If R is the electrical current loop radius:

$$r = \sqrt{R^2 + z^2}, \; \cos\alpha = \frac{R}{\sqrt{R^2 + z^2}}$$

$$B = \int \frac{\mu_o i R}{4\pi (R^2 + z^2)^{3/2}} dl$$

since $\int dl = 2\pi R$:

$$B = \frac{\mu_o i R^2}{2(R^2 + z^2)^{3/2}}$$

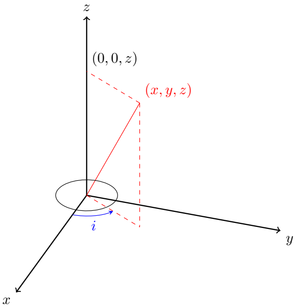
magnetic dipole

A magnetic dipole can be thought of as a very small radius electrical current loop. Prove to yourself that along the z axis, if $z >> R$:

$$B = \frac{\mu_o i R^2}{2z^3} = \frac{\mu_o i a}{2\pi z^3}$$

where a is the area of the small loop.

$\vec{B}$ off axis for a current loop



From Biot-Savart Law, we know that the magnitude of the magnetic field, $|\vec{B}|$, along the z-axis, normal to a current loop, is

$$B_z(z) = \frac{\mu_o ia}{2\pi z^3}$$

where μ_o is the magnetic permeability ($4\pi \times 10^{-7}$ N/amp^2), i is the current in the loop (amp), a is the area of the loop (m^2), which must be small relative to distance z (m).

More generally, with the x and y axes normal to z, in the plane of the current loop

$$B_x(x, y, z) = \frac{\mu_o}{4\pi} 3ia \frac{xz}{(x^2 + y^2 + z^2)^{5/2}}$$

$$B_y(x, y, z) = \frac{\mu_o}{4\pi} 3ia \frac{yz}{(x^2 + y^2 + z^2)^{5/2}}$$

$$B_z(x, y, z) = \frac{\mu_o}{4\pi} 3ia \frac{z^2 - \frac{1}{3}(x^2 + y^2 + z^2)}{(x^2 + y^2 + z^2)^{5/2}}$$

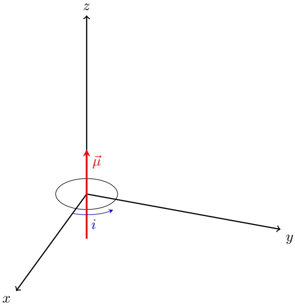
Magnetic field on the axis of the loop

Prove to yourself using the above equations that

$$B_z(0, 0, z) = \frac{\mu_o ia}{2\pi z^3}$$

in agreement with the derivation using the Biot-Savart law. What are ther values of $B_x(0, 0, z)$ and $B_y(0, 0, z)$?

Magnetic dipole moment



In practice, the term ia is replaced by a vector

$$\vec{\mu} = ia\vec{n}$$

where $\vec{n}$ is a unit vector directed in the positive z direction. $\vec{\mu}$ is called the magnetic moment, or the magnetic dipole moment. The units of the magnetic dipole moment are amp m.

The magnetic dipole moment can be expressed in terms of current flow in a loop (left), or in terms of the distance from one magnetic pole to another

$$\vec{\mu} = p\vec{l}$$

where p is the pole strength (amp m) and $\vec{l}$ is a vector pointing from one pole to the other (m). A third way to characterize magnetic dipole moment is particularly useful for solids, like magnets or magnetic rocks:

$$\vec{\mu} = \frac{1}{\mu_o}\vec{B}_r V$$

where μ_o is the magnetic permeability ($4\pi \times 10^{-7}$ N/amp^2), $\vec{B}_r$ is the residual field strength (T) (measured in a lab) and V is the volume of the magnetic sample (m^3).

Magnetic dipole moment

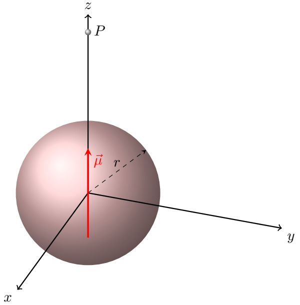
Prove to yourself that in all definitions, the units of the magnetic dipole moment are amp m^2.

Magnetization

The magnetization, $\vec{M}$, is the magnetic dipole moment per unit volume. Magnetization (amp/m) is used to estimate the magnetic dipole moment of an entire rock body. For a uniformly magnetized body

$$\vec{\mu} = \vec{M}V$$

where V is the volume of the magnetized body.



Example

Suppose a rock magnetics lab reports that the magnetization of a particular near Earth object (an ordinary chondrite) is 9 amp/m. If this body can be approximated as a uniformily magnetized sphere 50 m in radius, its magnetic dipole moment is

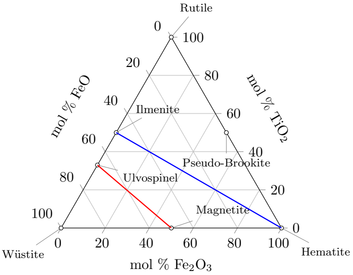
$$\vec{\mu} = \vec{M}\frac{4}{3}\pi r^3$$

where r is the radius of the sphere. The expected magnetic field due to the sphere at a point 1000 m from the center of the sphere along the z axis is

$$B_z(z) = \frac{\mu_o \vec{\mu}}{2\pi z^3}$$

Prove to yourself that the magnetic field strength at P(0,0,1000) is 0.94 nT, assuming there are no other external magnetic fields.

Magnetic minerals



The FeO-Fe_2O_3-TiO_2 ternary system. The solid solution for titanomagnetite is shown in red and the solid solution for titanohematite is shown in blue. For these solid solutions, Curie temperature increases from ilmenite/ulvöspinel to magnetite/hematite. Minerals lose their magnetic properties – becoming paramagnetic – at temperatures above the Curie temperature. The Curie temperature varies, being approximately 675 $^\circ$C for hematite and around 125 $^\circ$C for ilmenite. Therefore the mantle, magmas, and other hot rocks are paramagnetic and due not contribute to magnetic anomalies.

Most minerals are not magnetic (technically they are diamagnetic or paramagnetic) and so do not contribute to magnetization or to magnetic anomalies. Examples of minerals that do not contribute to magnetic anomalies are quartz and feldspar. Ferromagnetic and Ferrimagnetic minerals contribute to magnetic anomalies. Briefly, these minerals include magnetite, hematite, ilmenite, maghemite and ulvöspinel. All are characterized by a solid-solution between Fe and Ti, of the form $Fe_{3-x}Ti_xO_4$, where x is 0 or 1. Iron sulfides, such as pyrrhotite, are also significant contributors to rock magnetization, when they are present.

Magnetic minerals and magnetization

There is no simple relationship between magnetic mineralogy (the types of magnetic minerals in a rock, their percentages, their sizes) and the magnetization of the sample. Instead the vector of magnetization (its magnitude, inclination and declination) is determined through laboratory measurements performed on individual rock samples.

Susceptibility and remanent magnetization

Rocks containing magnetic minerals typically have two types of magnetization that each contribute to the magnetic dipole moment of the rock body. These two types of magnetization are induced and remanent magnetization.

Induced magnetization, $\vec{M}_i$, occurs when rocks are immersed in an external magnetic field, like the Earth's magnetic field. The magnitude of the induced magentization depends on both a proporationality constant, χ, called the magnetic susceptibility (which depends on the magnetic properties of the rock) and on the magnitude and direction of the external magnetic field.

$$\vec{M}_i = \chi \vec{H} = \frac{\chi \vec{B}_E}{\mu_o}$$

where $\vec{B}_E$ is the magntiude of the external field where the rock or rock body is located (amp) and μ_o is the magnetic permeability (T m /amp). $\vec{H}$ is the magnetization associated with the external field (amp/m). If there is no external magnetic field, $\vec{M}_i = 0$.

The higher the susceptibility, χ, the larger the resulting magnetic dipole moment, and hence the larger the magnetic anomaly. The induced magnetization vector M_i is almost always in the direction of the external magnetic field. Rarely, M_i is in the opposite dorection of the external magnetic field.

A rock sample's χ is determined using a magnetic susceptibility meter. A typical range of susceptibilities for basalts is $\chi = 5 \times 10^{-4}$ (SI) to 1×10^{-1} (SI) with a mean around 3×10^{-2} (SI) (χ is dimensionless and values are expressed in the SI system).

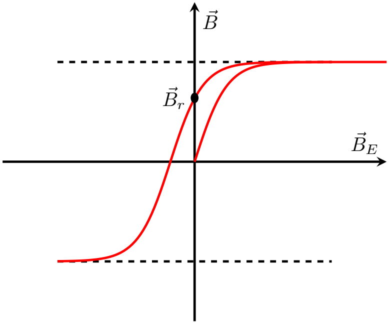
Susceptibility and remanent magnetization

Remanent magnetization, $\vec{M}_r$, is permanent magnetization and occurs in a rock whether the rock is currently immersed in an external magnetic field or not. Rocks acquire permanent magnetization by being immersed in a magnetic field at some point, often as they cool through the Curie temperature or as magnetic grains are deposited and consolidate into sedimentary rock.

The hysteresis curve shows how permanent magnetization is acquired by a rock sample. The axes of the hysteresis plot show the magnetization acquired by the rock sample ($\vec{B}$), as a function of the strength of the external magnetic field ($\vec{B}_E$). As a larger external field ($\vec{B}_E$) is applied, the rock acquires magnetization up to a saturation level (dashed line). Then as the magnitude of the external field is reduced, $\vec{B}$ decreases, but not at the same rate it increased. When $\vec{B}_E$ returns to zero, a permanent magnetization, $\vec{B}_r$, remains.

$$\vec{M}_r = \frac{\vec{B}_r}{\mu_o}$$

A negative external field must be applied for the sample to lose its remanent magnetization, which does not happen often in nature.



As the external magnetic field changes or disappears, $\vec{M}_r$ remains locked. Unlike induced magnetization, the magnitude and direction of $\vec{M}_r$ is not dependent on the current external field strength or direction.

Susceptibility and remanent magnetization

The total magnetization is the sum of induced and remanent magnetization vectors:

$$\vec{M} = \vec{M}_i + \vec{M}_r$$

The relative strength and directions of $\vec{M}_i$ and $\vec{M}_r$ affect the shapes of magnetic anomalies. On Earth:

- $\vec{M}_i$ is almost always in the direction of the current magnetic field.
- If $\vec{M}_r$ is approximately in the direction of the current magnetic field the rock is *normally* magnetized.
- If $\vec{M}_r$ is approximately in opposite the direction of the current magnetic field the rock is *reversely* magnetized, and very likely formed during a polarity reversal in the Earth's magnetic field.
- Tectonic rotations (folding, faulting) change the orientation of $\vec{M}_r$ with respect to the external field.

The effect of these differences depends on the relative magnitudes of $\vec{M}_r$ and $\vec{M}_i$. For example, basalts carry remanent magnetization of 1–100 $\mathrm{A\,m}^{-1}$. This means that for basalts (and many igneous rocks) the remanent magnetization is the dominant contributor to the total vector of magnetization, and to the magnetic field anomaly. Basalts and similar igneous rocks have high *Koenigsberger* ratios, Q:

$$Q = \frac{\vec{M}_r}{\vec{M}_i}$$

Answer the following questions to make sure you understand the main concepts in this module

1. A 3-axis fluxgate magnetometer reports the components of the magnetic field $(B_x, B_y, B_z) = (13000 \text{ nT}, 546 \text{ nT}, 30450 \text{ nT})$ with x pointing north, y east, and z down. Calculate the magnitude of $\vec{B}$, its inclination and declination, and the magnitude of the horizontal component of $\vec{B}$.
2. An electrical wire runs E-W, carries a 100 amp current (flowing west), and is buried 1 m beneath the ground surface. The regional magnetic field, $\vec{B}_E$, is horizontal and points North. (a) What is the expected magnitude, B, directly over the wire measured at the ground surface (use Ampere's Law)? (b) What are the expected magnitudes of B_x, B_y and B_z? (c) Sketch the shape of the expected change in magnitude $|\vec{B}_E + \vec{B}|$, as magnetic values are determined from S to N across the buried wire. (d) How does the expected shape of the magnetic anomaly (variation in magnitude of B) change along the same profile if $\vec{B}_E$ points down? Draw a sketch of the relative change in $|\vec{B}_E + \vec{B}|$ with $\vec{B}_E$ pointing down.
3. A cube of rock, 10 m on a side, is centered at point $O(0, 0, 0)$ and has a measured magnetization of 3 amp/m in the direction of the positive z-axis. What is the expected magnitude and direction of the magnetic field strength, B, at point $P(30, 30, 75)$?
4. A magnetometer measures the strength of the magnetic field, $B = 62000$ nT at the magnetic north pole, on the surface of the Earth. Assume the Earth's radius at the magnetic north pole is 6357000 m. Assume that the entire Earth's magnetic field originates in the core and is well approximated by a loop current. Calculate the magentic dipole moment, μ, for Earth and estimate the magnetization, $\vec{M}$, of the core.